Example 4.6.1: Curve fitting.

Find the equation for a parabola that passes through $(3, 25)$, $(1, 5)$, and $(-2, 20)$.

Solution

The general form for a parabola is $y = ax^2 + bx + c$. Substituting the x- and y- coordinates of each point into the general form yields the system below.

-2 * -2

$$\begin{aligned} 9a + 3b + c &= 25 & (3, 25) \\ 4a - 2b + c &= 20 & (-2, 20) \\ a + b + c &= 5 & (1, 5) \end{aligned}$$

The system is expressed as an augmented matrix.

$$\left[\begin{array}{ccc|c} 9 & 3 & 1 & 25 \\ 4 & -2 & 1 & 20 \\ 1 & 1 & 1 & 5 \end{array}\right]$$

Forward elimination is performed to transform the augmented matrix into echelon form.

A B C

$$\left[\begin{array}{ccc|c} 9 & 3 & 1 & 25 \\ 4 & -2 & 1 & 20 \\ 1 & 1 & 1 & 5 \end{array}\right] \quad \cdots \quad \xrightarrow{R_3 - R_2 \to R_3} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 0 & -6 & -3 & 0 \\ 0 & 0 & -5 & -20 \end{array}\right]$$

The corresponding system of linear equations is

$$\begin{aligned} a + b + c &= 5 \\ -6b - 3c &= 0 \\ -5c &= -20. \end{aligned}$$

1

Following forward elimination, back-substitution is performed beginning with the last equation.

$$\begin{aligned} -5c &= -20 \\ c &= 4 \end{aligned}$$

1

Next, $c = 4$ is substituted into the second equation to find b.

$$\begin{aligned} 6b + 3c &= 0 \\ -6b - 3(4) &= 0 \\ -6b &= 12 \\ b &= -2 \end{aligned}$$

2

Finally, $b = -2$ and $c = 4$ are substituted into the first equation to find a.

$$\begin{aligned} a + b + c &= 5 \\ a + (-2) + 4 &= 5 \\ a + 2 &= 5 \\ a &= 3 \end{aligned}$$

3

Thus, the equation for the parabola that passes through $(3, 25)$, $(1, 5)$, and $(-2, 20)$ is given by the equation $y = 3x^2 - 2x + 4$.

Analysis

You are correct. Based on the first image, the first step of forward elimination is to find the **current pivot** in the first column, which is defined as the first nonzero entry. In your initial matrix, the first entry in the first column is a `9`, so it should be the pivot.

The solution in the second image made a mistake. It swapped the first row with the third row for no apparent reason, as there was no zero in the pivot position that needed to be moved. Swapping rows is a valid elementary row operation, but it's only necessary if the pivot is a zero.

Let's walk through the **correct** first step of forward elimination for the matrix you provided:

$$\begin{bmatrix} 9 & 3 & 1 & 25 \\ 4 & -2 & 1 & 20 \\ 1 & 1 & 1 & 5 \end{bmatrix}$$

1. **Identify the pivot:** The first nonzero entry in the first column is the `9` in the first row. This is our pivot.
2. **Make entries below the pivot zero:** We need to use row operations to make the `4` and `1` in the first column become zero.
    - To make the `4` zero, we can perform the operation: $R_2 - \frac{4}{9}R_1 \to R_2$
    - To make the `1` zero, we can perform the operation: $R_3 - \frac{1}{9}R_1 \to R_3$

The result of these operations would be a new matrix where the first column has zeros below the `9`. The solution provided in the image skips this logical first step and swaps the rows. While this doesn't change the final solution to the system of equations, it does not follow the standard, most straightforward procedure for forward elimination.